

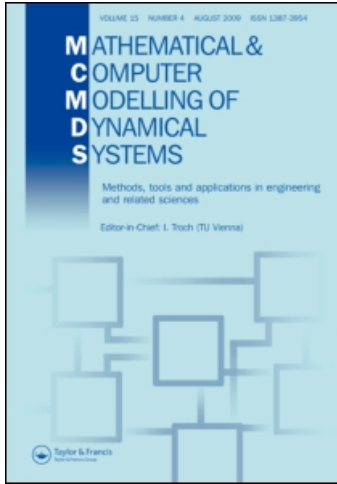
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## Mathematical and Computer Modelling of Dynamical Systems

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### A dynamical model for describing behavioural interventions for weight loss and body composition change

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## A dynamical model for describing behavioural interventions for weight loss and body composition change

J.-Emeterio Navarro-Barrientos<sup>a†</sup>, Daniel E. Rivera<sup>a\*</sup> and Linda M. Collins<sup>b</sup>

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We present a dynamical model incorporating both physiological and psychological factors that predict changes in body mass and composition during the course of a behavioural intervention for weight loss. The model consists of a three-compartment energy balance integrated with a mechanistic psychological model inspired by the Theory of Planned Behaviour. This describes how important variables in a behavioural intervention can influence healthy eating habits and increased physical activity over time. The novelty of the approach lies in representing the behavioural intervention as a dynamical system and the integration of the psychological and energy balance models. Two simulation scenarios are presented that illustrate how the model can improve the understanding of how changes in intervention components and participant differences affect outcomes. Consequently, the model can be used to inform behavioural scientists in the design of optimized interventions for weight loss and body composition change.

**Keywords:** behavioural interventions; weight loss; obesity; body composition; energy balance; Theory of Planned Behaviour; dynamical systems

### 1. Introduction

Obesity rates in the United States have increased substantially in recent decades [1]. In 2000, the percentage of adults in the United States with body mass index (BMI) exceeding 30 was 19.8% [2]. The 2000 census reported that 27% of US adults do not engage in any physical activity, and only 24.4% of US adults consumed at least five servings of fruits and vegetables a day. Among the US adults participating in programs for losing or maintaining weight, only 17.5% were following the recommended guidelines for reducing calories and increasing physical activity [2]. More recently, the World Health Organization has revealed that 2.7 and 1.9 million deaths per year are attributable to low fruit and vegetable intake and low physical activity, respectively [3]. Unhealthy diet behaviours are responsible for 31% of the cases of ischemic heart disease, 11% of the cases of strokes and 19% of the cases of gastrointestinal cancer.

Because obesity represents a preventable cause of premature morbidity and mortality, much research activity has been devoted to understanding its causes, and a number of diverse solutions have been proposed. Some of these have major disadvantages; for instance,

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bariatric surgery and very low calorie diets usually lead to a loss of primarily fat-free mass, which in turn causes a temporary decrease on energy requirements and in the long-run a possible regain of the weight lost [4,5]. Solutions leading to permanent weight loss require sustained lifestyle changes in an individual; consequently, developing optimized behavioural interventions that promote healthy eating habits and increased physical activity represents a problem of both fundamental and practical importance.

The primary goal of this article is to improve the understanding of behavioural weight change interventions by expressing these as dynamical systems. Dynamic modelling considers how important system variables (e.g. intervention outcomes) respond to changes in input variables (e.g. intervention dosages, exogenous influences) over time. A dynamical model can be used to answer questions regarding what variables to measure and how often and the speed and functional form of the outcome responses as a result of decisions regarding the timing, spacing and dosage levels of intervention components. A dynamical systems approach has been proposed in the analysis of novel behavioural interventions, for instance, adaptive interventions for behavioural health [6].

To achieve this goal, we develop in this article a dynamical model for daily weight change incorporating both physiological and psychological considerations. For the physiological component, we rely on the concept of energy balance to obtain a model that describes the net effect of energy intake from food minus energy consumption, the latter including physical activity. This model can be used to determine the reduction in caloric intake and the level of physical activity that are necessary to achieve a desired weight loss goal. For the psychological component, we present a model for the dynamics of diet and exercise behaviour. This model explains how intentions, subjective norms, attitudes and other system variables that may be impacted by an intervention can result in healthy eating habits and increased physical activity over time. A model based on the widely accepted Theory of Planned Behaviour (TPB; [7]) is used for this purpose. Figure 1 shows the general conceptual diagram for the integrated dynamical model philosophy developed in this article.

The article is organized as follows. Section 2 presents the energy balance model. Section 3 gives a brief description of the TPB and presents a mechanistic dynamical model for the TPB based on fluid analogies. Section 4 describes two representative simulations of the dynamical model and discusses the role and importance of some of the parameters in the model. Finally, Section 5 summarizes our main conclusions and discusses the areas of current and further study.

## 2. Energy balance model

The functional relationship between energy balance and weight change has been studied extensively in the literature [8–18]. In this section, the goal is to develop a simple yet informative input–output energy balance model that will serve as the basis for describing how weight varies because of changes in diet and physical activity. First, we present the basic dynamics governing energy balance using a conventional two-compartment model; this is

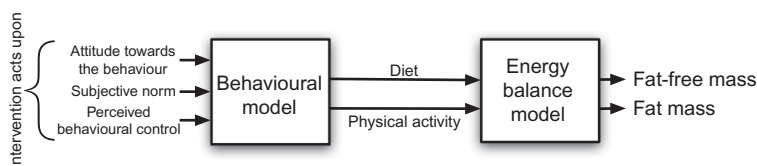


Figure 1. General diagram of the dynamical model for body mass and composition change.

followed by a more complete three-compartment model. The three-compartment model is validated using data from the Minnesota semi-starvation experiment [19].

**2.1. Basic dynamics and two-compartment model**

The normal daily energy balance  $EB(t)$  is described as follows:

$$EB(t) = EI(t) - EE(t), \tag{1}$$

where  $EI(t)$  is the energy intake and  $EE(t)$  is the energy expenditure at day  $t$ . The daily energy intake  $EI$ , expressed in kilocalories (kcal), can be modelled based on the daily energy requirements and dietary reference intakes [20,21]. However, for simplicity we model  $EI$  using the Atwater methods of energy calculation resulting from carbohydrate intake (CI), fat intake (FI) and protein intake (PI), all expressed in units of g/day [22,23]:

$$EI(t) = a_1CI(t) + a_2FI(t) + a_3PI(t), \tag{2}$$

where  $a_1 = 4$  kcal/g,  $a_2 = 9$  kcal/g and  $a_3 = 4$  kcal/g.

The daily energy expenditure  $EE$ , expressed in kcal, is calculated as follows:

$$EE(t) = TEF(t) + PA(t) + RMR(t), \tag{3}$$

where the thermic effect of feeding  $TEF(t)$  denotes the energy expended in processing food,  $PA(t)$  the energy spent as a result of physical activity and  $RMR(t)$  the resting metabolic rate. The energy expended on  $TEF$  usually ranges from 7 to 15% of the total energy intake. The thermic effect of exercise  $PA$ , expressed in kcal, captures the energy consumed as a result of conducting work activities, household tasks and physical exercise. The  $RMR$  refers to the energy needed to maintain basic physiological processes and is best assessed in an overnight fasted state. The  $RMR$  represents a substantial percentage (45–70%) of energy expenditure for the typical individual [20].

In the two-compartment model, the total body mass (BM) is given by the sum of two compartments corresponding to fat mass (FM) and fat-free mass (FFM):

$$BM(t) = FM(t) + FFM(t). \tag{4}$$

The daily energy balance in Equation (1) is partitioned into one of these two compartments, each described by its own differential equation,

$$\frac{dFM(t)}{dt} = \frac{(1 - p(t))EB(t)}{\rho_{FM}}, \tag{5}$$

$$\frac{dFFM(t)}{dt} = \frac{p(t)EB(t)}{\rho_{FFM}}. \tag{6}$$

$\rho_{FM} = 9400$  kcal/kg and  $\rho_{FFM} = 1800$  kcal/kg are energy densities, whereas  $p(t)$  is the  $p$ -ratio. The  $p$ -ratio is the parameter that assigns a percentage of the imbalance denoted by  $EB$  to the compartments  $FM$  and  $FFM$ , respectively [24]. The works of Westerterp *et al.* [10] and Chow and Hall [14] provide illustrations of two-compartment energy balance models. Hall [25] was the first to define  $p$  as given by the Forbes formula [26]:

$$p = \frac{C}{(C + FM)}; \quad C = 10.4 \frac{\rho_{FFM}}{\rho_{FM}}. \tag{7}$$

## 2.2. Three-compartment model

### 2.2.1. Model dynamics

Recent work by Hall and Chow [5,13,14,17,18] enables the traditional two-compartment model to be extended into a three-compartment model in which fat-free mass is further divided into lean mass (LM) and extracellular fluid (ECF). The total body mass is given by the sum of these three compartments as follows:

$$BM(t) = FM(t) + \overbrace{LM(t) + ECF(t)}^{FFM(t)}. \quad (8)$$

Each of these is defined by its own differential equation:

$$\frac{dFM(t)}{dt} = \frac{(1 - p(t))EB(t)}{\rho_{FM}}, \quad (9)$$

$$\frac{dLM(t)}{dt} = \frac{p(t)EB(t)}{\rho_{LM}}, \quad (10)$$

$$\frac{dECF(t)}{dt} = \frac{\rho_w}{[Na]} \left( \Delta Na_{\text{diet}} - \xi_{Na}(ECF - ECF_{\text{init}}) - \xi_{CI} \left( 1 - \frac{CI}{CI_b} \right) \right). \quad (11)$$

The lean mass energy density  $\rho_{LM}$  is equivalent to  $\rho_{FFM} = \rho_{LM} = 1800$  kcal/kg. Likewise,  $p$  is given as in Equation (7), but with  $C$  determined on the basis of the lean mass

$$C = 10.4 \frac{\rho_{LM}}{\rho_{FM}}. \quad (12)$$

For the extracellular fluid as Equation (11),  $\Delta Na_{\text{diet}}$  is the change on sodium in mg/d,  $CI_b$  is the baseline carbohydrate intake,  $[Na] = 3.22$  mg/ml,  $\xi_{Na} = 3$  mg/ml/d and  $\xi_{CI} = 4000$  mg/d.  $\rho_w$  is the density of water, which we consider throughout as 1 kg/l (1 mg/ml).

The general expression for energy expenditure EE in Equation (3) is modelled explicitly as follows:

$$EE = \beta EI + \delta BM + K + \gamma_{LM}LM + \gamma_{FM}FM + \eta_{FM} \frac{dFM}{dt} + \eta_{LM} \frac{dLM}{dt}. \quad (13)$$

$\beta = 0.24$  is the coefficient for the thermic effect of feeding (TEF).  $\delta$ , the physical activity coefficient, is expressed in terms of kcal/kg of body mass.  $\gamma_{LM} = 22$  kcal/kg/d,  $\gamma_{FM} = 3.2$  kcal/kg/d,  $\eta_{FM} = 180$  kcal/kg and  $\eta_{LM} = 230$  kcal/kg are the coefficients for the calculation of the resting metabolic rate (RMR). The constant  $K$  accounts for initial conditions and is determined by solving Equation (1) assuming an initial steady state ( $\overline{EI} - \overline{EE} = 0$ , with  $d\overline{FM}/dt = d\overline{LM}/dt = 0$  by definition of steady state):

$$K = -\gamma_{LM}\overline{LM} - \gamma_{FM}\overline{FM} - \delta\overline{BM} + \overline{EI}(1 - \beta). \quad (14)$$

The steady state is denoted by a bar over any time-dependent variable. By substituting Equations (9) and (10) in Equation (13), it becomes possible to obtain a closed-form expression for energy expenditure without the need for derivatives of FM and LM:

$$EE = \frac{\rho_{FM} \rho_{LM} (\delta \overline{BM} + \gamma_{FM} \overline{FM} + \gamma_{LM} \overline{LM} + K) + EI (\eta_{LM} p \rho_{FM} + \rho_{LM} (\eta_{FM} - \eta_{FM} p + \beta \rho_{FM}))}{\eta_{LM} p \rho_{FM} + \rho_{LM} (\eta_{FM} - \eta_{FM} p + \rho_{FM})}. \quad (15)$$

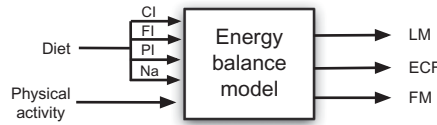


Figure 2. Input–output block diagram representation for the three-compartment energy balance model. Primary inputs to the model consist of physical activity and diet. Diet in turn is composed of carbohydrate intake (CI), fat intake (FI), protein intake (PI) and sodium intake (Na); the output compartments consist of lean mass (LM), fat mass (FM), and extracellular fluid (ECF).

In summary, the three-compartment dynamical model consists of CI, FI, PI,  $\Delta Na_{\text{diet}}$  and  $\delta$  as inputs, and FM, LM and ECF as outputs whose sum corresponds to the total body mass, BM. A block diagram is presented in Figure 2.

### 2.2.2. Initialization

Initial conditions for each of the compartments may be known experimentally; if that is not the case, these can be estimated based on various correlations available in the literature. For example, one can compute the initial lean mass LM using the regression formulas from Westerterp *et al.* [10]:

$$LM(\text{Men}) = -18.36 - 0.105 \text{ Age (years)} + 34.009 \text{ Height (m)} + 0.292 \text{ BM}, \quad (16)$$

$$LM(\text{Women}) = -12.47 - 0.074 \text{ Age (years)} + 27.392 \text{ Height (m)} + 0.218 \text{ BM}. \quad (17)$$

The initial fat mass FM can be estimated from the regression equations from Jackson *et al.* [27]:

$$FM(\text{Men}) = \frac{3.76 (\text{BMI})(\text{BM}) - 0.04 (\text{BMI})^2 (\text{BM}) - 47.80 \text{ BM}}{100}, \quad (18)$$

$$FM(\text{Women}) = \frac{4.35 (\text{BMI})(\text{BM}) - 0.05 (\text{BMI})^2 (\text{BM}) - 46.24 \text{ BM}}{100}. \quad (19)$$

The previous two equations require knowledge of the initial body mass BM and body mass index (BMI), which is determined as follows:

$$\text{BMI} = \text{BM} / (\text{Height(m)})^2. \quad (20)$$

To determine the initial ECF, we use the regression equations of Silva *et al.* [28] for men and women, respectively:

$$\text{ECF}(\text{Men}) = 0.025 \times \text{Age} + 9.57 \times \text{Height} + 0.191 \times \text{BM} - 12.4; \quad (21)$$

$$\text{ECF}(\text{Women}) = 5.98 \times \text{Height} + 0.167 \times \text{BM} - 4. \quad (22)$$

Silva *et al.* [28] noted that ECF was strongly affected by weight and height in both men and women. However, age made significant contributions to the models for men but not for women.

### 2.2.3. Validation with the Minnesota semi-starvation experiment

A well-known experimental study in which daily weight change, diet and activity were tracked for a group of participants is the Minnesota semi-starvation experiment [19]. The

goal of this study was to determine the physiological and psychological effects of weight loss and weight gain diets on human beings. The experiment was performed on 32 healthy men with the following average characteristics: age = 25.5 years, height = 1.63 m, BM = 69.39 kg and FM = 9.0 kg. Various researchers have compared mechanistic models against this experiment, showing good agreement [13,29]. Table 1 summarizes the time line and diets applied to the participants during the experiment.

Based on the average characteristics of the participants, we used Equation (22) to determine the initial ECF (17.09 kg) and Equation (8) to find the initial LM (43.29 kg). For calculating ECF, we assume a pre-intervention Na of 4000 mg/d that comprises a typical diet. The Minnesota experiment reports that the average salt intake during the trial consisted of 12.12 g of NaCl; given that NaCl consists of 39.337% Na, this implies a Na intake of 4767 mg/day, resulting in  $\Delta Na_{\text{diet}} = 767$  mg/d as an input to the model. This change is kept constant throughout the period of the trial.

Physical activity during the Minnesota experiment consisted of approximately 15 hours per week working in either maintenance of the laboratory and living quarters, laundry, laboratory assistance or shop duties or clerical and statistical work; walking outdoors 22 miles per week; spending a half-hour per week on a motor-driven treadmill at 3.5 miles per hour on a 10% grade and walking 2–3 miles per day to and from the dining hall. Hall [13] interpreted the report in Keys *et al.* [19] of an observed decrease in physical activity and a reduced ‘activity drive’ during the semi-starvation period to signify that the PA coefficient  $\delta$  should decrease with time during the period of starvation, then increase as refeeding takes place. To this effect, Hall and Jordan [5] considered a maximum value of  $\delta = 24.5$  kcal/kg/d and a minimum value of  $\delta = 8$  kcal/kg/d and a minimum value of  $\delta = 8$  kcal/kg/d during the course of the experiment; we rely on these values in defining the input profile for our model.

Figure 3 compares the results of the proposed three-compartment model with the experimental data and the two-compartment model of Westerterp *et al.* [10]. Changes in EI summarized in Table 1 as well as the changes in  $\delta$  and  $Na_{\text{diet}}$  previously reported serve as inputs to the energy balance model. Figure 3 shows a good agreement between the average body weight of the participants in the Minnesota experiment against the three-compartment simulation result through the duration of the study. The three-compartment model displays a superior fit compared with the model from Westerterp *et al.* [10].

Additional analysis and simulation results on the Minnesota experiment, including subgroup results during the rehabilitation phase, can be found in the technical report by Navarro-Barrientos and Rivera [30]. The energy balance model can be evaluated interactively using the software package *Weigh-IT* developed in the ASU-Control Systems Engineering Laboratory (<http://cseel.asu.edu/Weigh-IT>).

Table 1. Time line and diets of the Minnesota semi-starvation experiment.

Time line	Diet
Control diet (12 weeks)	Average basic diet provided 3492 kcal/d: PI = 112 g, FI = 124 g and CHO = 482.
Semi-starvation diet (24 weeks)	Average daily intake was 1570 kcal/d: PI = 50 g, FI = 30 g and CI = 275 g.
Rehabilitation average intake (12 weeks restricted + 8 weeks unrestricted)	1–6th week = 2449 kcal/d. 7–10th week = 3257 kcal/d. 11–12th week = 3518 kcal/d. 13–20th week = 3200–4500 kcal/d.

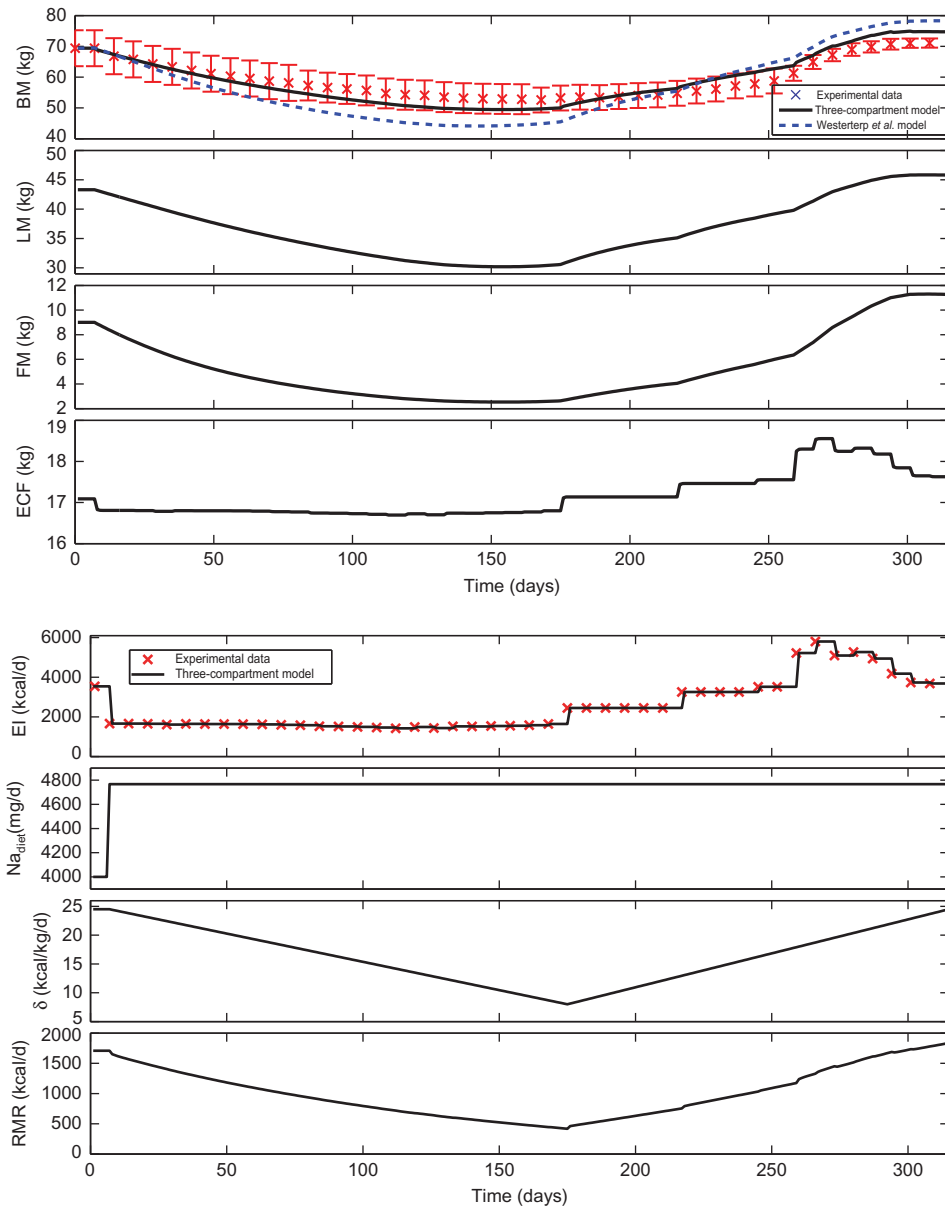


Figure 3. Body mass BM and compartments (lean mass (LM), fat mass (FM) and extracellular fluid (ECF)) obtained from the three-compartment model for the Minnesota semi-starvation experiment [19]. Body mass is compared to experimental data ( $\times$ , with error bars) and the total body mass from the Westerterp *et al.* [10] two-compartment model (dashed line).

### 3. Behavioural Model

In addition to capturing the dynamics of energy balance, our approach requires modelling the psychology of diet and exercise behaviour in human beings. In this section a relevant psychological theory (the TPB) is extended into a dynamical system representation that can



be integrated with the energy balance model to provide a comprehensive behavioural intervention model. A mechanistic modelling framework relying on a fluid analogy and patterned after concepts in inventory management in supply chains will be used for this purpose.

### 3.1. Theory of planned behaviour

For almost two decades, the TPB [7] has been used extensively within the social sciences for describing the relationship between behaviours, intentions, attitudes, norms and perceived control. Many studies have relied on the TPB and its forerunner, the Theory of Reasoned Action (TRA) [31] to describe the mechanisms of behaviour in diverse application settings; these include healthy eating habits [32] and exercise [33–35]. In the TPB framework, intention is an indication of the readiness of a person to perform a given behaviour, where as behaviour is an observable response in a given situation with respect to a given target. Intention is influenced by the following components:

- *Attitude Towards the Behaviour*: This is the degree to which performing the behaviour is positively or negatively valued. It is determined by the *strength of beliefs about the outcome* and the *evaluation of the outcome*.
- *Subjective Norm*: This is the perceived social pressure to engage or not engage in a behaviour. It is determined by the strength of the beliefs about what people want the person to do, also called *normative beliefs*, and the desire to please people, also called *motivation to comply*.
- *Perceived Behavioural Control (PBC)*: This reflects the perception of the ability to perform a given behaviour, that is, the beliefs about the presence of factors that may facilitate or impede the performance of the behaviour. It is determined by the *strength of each control belief* and the *perceived power of the control factor*.

A standard mathematical representation for TPB relies on Structural Equation Modelling (SEM) [36]. The field of SEM is substantial, but in this work we limit ourselves to a special case of SEM called path analysis. The main characteristics of path analysis models is that they do not contain latent variables, that is, all problem variables are observed, and the independent variables are assumed to have no measurement error [37]. The TPB represented as a path analysis model with a vector  $\eta$  of endogenous variables and a vector  $\xi$  of exogenous variables is expressed as follows:

$$\eta = \mathbf{B} \eta + \Gamma \xi + \zeta, \quad (23)$$

$$\begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \beta_{41} & \beta_{42} & \beta_{43} & 0 & 0 \\ 0 & 0 & \beta_{53} & \beta_{54} & 0 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{bmatrix} + \begin{bmatrix} \gamma_{11} & 0 & 0 \\ 0 & \gamma_{22} & 0 \\ 0 & 0 & \gamma_{33} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} + \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{bmatrix}, \quad (24)$$

where  $\mathbf{B}$  and  $\Gamma$  are matrices of  $\beta_{ij}$  and  $\gamma_{ij}$  regression weights, respectively, and  $\zeta$  is a vector of disturbance variables. Figure 4 shows the intention-behaviour TPB path analysis model for Equation (24). Typically, the principles of TPB assume that the attitude towards the behaviour  $\eta_1$ , the subjective norm  $\eta_2$  and the PBC  $\eta_3$  are estimated using the expectancy-value model, which considers the sum over the person's behavioural beliefs, normative

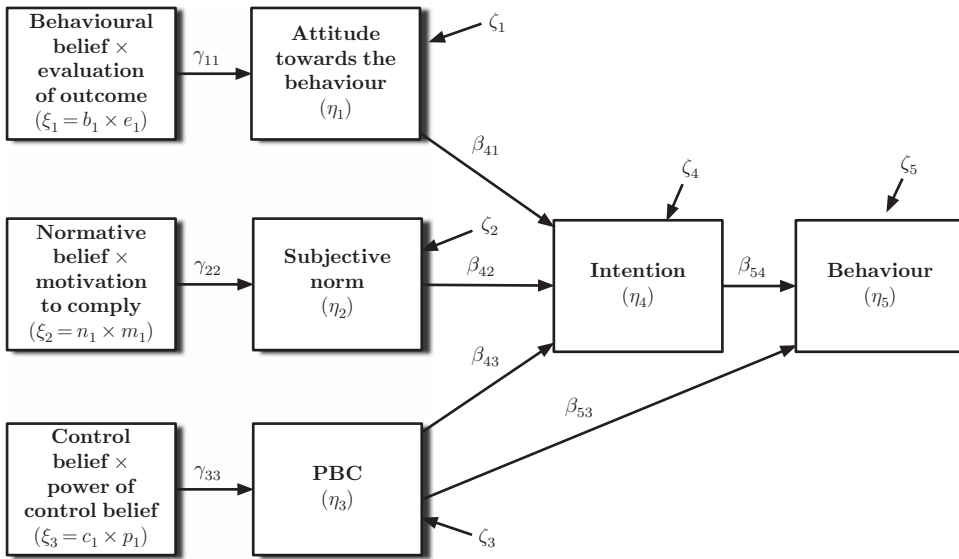


Figure 4. Path diagram for the TPB with three exogenous variables  $\xi_i$ , five endogenous variables  $\eta_i$ , regression weights  $\gamma_{ij}$  and  $\beta_{ij}$  and disturbances  $\zeta_i$ . PBC stands for perceived behavioural control.

beliefs and control beliefs, respectively, that are accessible at the time. However, for simplicity and without loss of generality, we consider only one exogenous variable per compartment. Thus,

$$\xi_1 = b_1 \times e_1 \tag{25}$$

= strength of beliefs about the outcome × evaluation of the outcome ,

$$\xi_2 = n_1 \times m_1 \tag{26}$$

= strength of normative beliefs × motivation to comply ,

$$\xi_3 = c_1 \times p_1 \tag{27}$$

= strength of control belief × perceived power of the control factor .

The exogenous variables  $\xi_1$ ,  $\xi_2$  and  $\xi_3$  and their subcomponents form the basis for the inputs of a dynamical model for the TPB; this is explained in more detail in the ensuing subsection.

### 3.2. Dynamic fluid analogy for the TPB

The classical SEM–TPB model, as expressed in Equation (23), represents a static (i.e. steady state) system that does not capture any changing behaviour over time. To expand the TPB model to include dynamic effects, we propose the use of a fluid analogy that parallels the problem of inventory management in supply chains [38]. This analogy is expressed diagrammatically in Figure 5. We consider a dynamic fluid analogy of TPB with five *inventories*: attitude  $\eta_1$ , subjective norm  $\eta_2$ , PBC  $\eta_3$ , intention  $\eta_4$  and behaviour  $\eta_5$ . Each inventory is replenished by inflow streams and depleted by outflow streams. The path diagram model

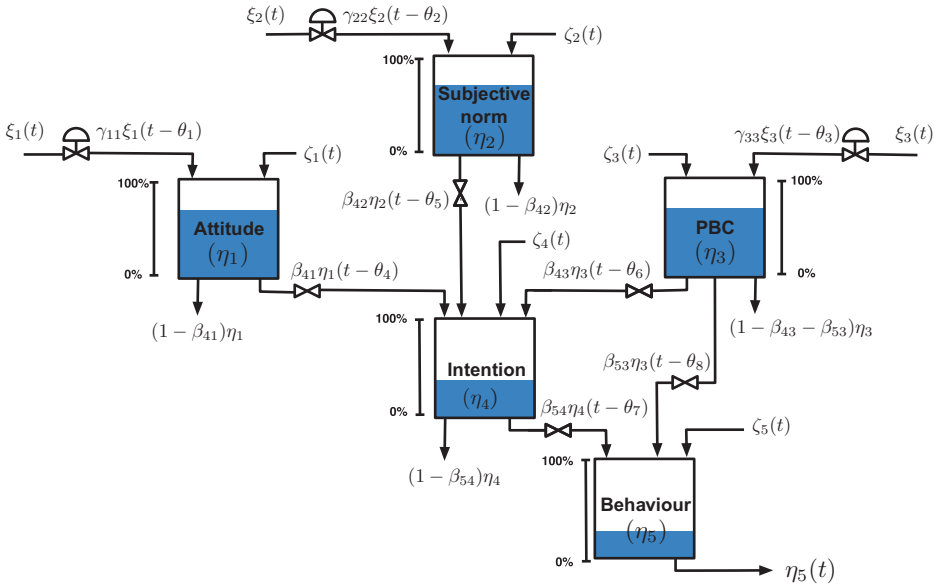


Figure 5. Fluid analogy for the TPB corresponding to the path diagram depicted in Figure 4. PBC stands for perceived behavioural control.

coefficients  $\gamma_{11}, \dots, \gamma_{33}$  are the inflow resistances and  $\beta_{41}, \dots, \beta_{54}$  are the outflow resistances, which can be physically interpreted as those fractions of the inventories of the system that serve as inflows to the subsequent layer in the path analysis model.

To generate the dynamical system description, we apply the principle of conservation of mass to each inventory, where accumulation corresponds to the net difference between the mass inflows and outflows:

$$\text{Accumulation} = \text{Inflow} - \text{Outflow}. \tag{28}$$

Relying on the rate form for Equation (28) leads to a system of differential equations according to:

$$\begin{aligned} \tau_1 \frac{d\eta_1}{dt} &= \gamma_{11}\xi_1(t - \theta_1) - \beta_{41}\eta_1(t) - (1 - \beta_{41})\eta_1(t) + \zeta_1(t) \\ &= \gamma_{11}\xi_1(t - \theta_1) - \eta_1(t) + \zeta_1(t), \end{aligned} \tag{29}$$

$$\begin{aligned} \tau_2 \frac{d\eta_2}{dt} &= \gamma_{22}\xi_2(t - \theta_2) - \beta_{42}\eta_2(t) - (1 - \beta_{42})\eta_2(t) + \zeta_2(t) \\ &= \gamma_{22}\xi_2(t - \theta_2) - \eta_2(t) + \zeta_2(t), \end{aligned} \tag{30}$$

$$\begin{aligned} \tau_3 \frac{d\eta_3}{dt} &= \gamma_{33}\xi_3(t - \theta_3) - \beta_{43}\eta_3(t) - \beta_{53}\eta_3(t) - (1 - \beta_{43} - \beta_{53})\eta_3(t) + \zeta_3(t) \\ &= \gamma_{33}\xi_3(t - \theta_3) - \eta_3(t) + \zeta_3(t), \end{aligned} \tag{31}$$

$$\begin{aligned} \tau_4 \frac{d\eta_4}{dt} &= \beta_{41}\eta_1(t - \theta_4) + \beta_{42}\eta_2(t - \theta_5) + \beta_{43}\eta_3(t - \theta_6) - \beta_{54}\eta_4(t) - (1 - \beta_{54})\eta_4 + \zeta_4(t) \\ &= \beta_{41}\eta_1(t - \theta_4) + \beta_{42}\eta_2(t - \theta_5) + \beta_{43}\eta_3(t - \theta_6) - \eta_4(t) + \zeta_4(t), \end{aligned} \tag{32}$$

$$\tau_5 \frac{d\eta_5}{dt} = \beta_{54}\eta_4(t - \theta_7) + \beta_{53}\eta_3(t - \theta_8) - \eta_5(t) + \zeta_5(t), \tag{33}$$

where  $\xi_1(t) = b_1(t)e_1(t)$ ,  $\xi_2(t) = n_1(t)m_1(t)$  and  $\xi_3(t) = c_1(t)p_1(t)$  according to Equations (25)–(27), and  $\zeta_1, \dots, \zeta_5$  are zero-mean stochastic signals. The dynamical system representation according to Equations (29)–(33) includes all the SEM model parameters and is enhanced by the presence of *time delays*  $\theta_1, \dots, \theta_8$  that model transportation lags between the inflows and outflows, and *time constants*  $\tau_1, \dots, \tau_5$  that capture the capacity of the tanks for each inventory in the system, and allow for exponential decay (or growth) in the system variables. These parameters can be used to determine the speed at which an intervention participant or population can transition between values for  $\eta_1, \dots, \eta_5$  as a result of changes in  $\xi_1, \xi_2$  and  $\xi_3$ . A number of important points of interest are summarized below:

- (1) At steady state, that is, when  $\frac{d\eta_i}{dt} = 0$ , Equations (29)–(33) reduce to the SEM model according to Equation (23) without approximation.
- (2) The SEM model coefficients  $\gamma_{ij}$  and  $\beta_{ij}$  correspond directly to gains in the dynamical system.
- (3) Because the PBC inventory feeds both the downstream intention and the behaviour inventories, the outflow resistances from PBC are subject to the constraint:

$$\beta_{53} + \beta_{54} \leq 1. \tag{34}$$

- (4) The initial level of the inventories, which are determined by finding the solution to the system of Equations (29)–(33) at steady state, corresponds to:

$$\bar{\eta}_1 = \gamma_{11}\xi_1, \tag{35}$$

$$\bar{\eta}_2 = \gamma_{22}\xi_2, \tag{36}$$

$$\bar{\eta}_3 = \gamma_{33}\xi_3, \tag{37}$$

$$\bar{\eta}_4 = \beta_{41}\bar{\eta}_1 + \beta_{42}\bar{\eta}_2 + \beta_{43}\bar{\eta}_3, \tag{38}$$

$$\bar{\eta}_5 = \beta_{54}\bar{\eta}_4 + \beta_{53}\bar{\eta}_3. \tag{39}$$

- (5) The dynamical model description does not require (or assume) a single subject interpretation. The SEM model is naturally estimated cross-sectionally from the data obtained from a population; the dynamical model can be estimated with respect to a population as well but will require availability of repeated measurements over time.

The dynamical model representation according to Equations (29)–(33) can be extended with higher order derivatives. This enables including additional parameters that can generate a greater diversity of dynamical system responses, such as underdamped and inverse response. Using the case of the attitude inventory as an example, it is possible to rewrite Equation (29) as

$$\tau_1^2 \frac{d^2\eta_1}{dt^2} + 2\zeta\tau_1 \frac{d\eta_1}{dt} = \gamma_{11} \left( \xi_1(t - \theta_1) + \tau_a \frac{d\xi_1(t - \theta_1)}{dt} \right) - \eta_1(t) + \zeta_1(t). \quad (40)$$

The parameter  $\zeta$  indicates a damping coefficient that can be used to define overdamped ( $\zeta > 1$ ), critically damped ( $\zeta = 1$ ) or underdamped ( $0 \leq \zeta < 1$ ) system responses, whereas  $\tau_a$  can be used to introduce model zeros that can lead to non-minimum phase system phenomena such as inverse response ( $\tau_a < 0$ ). An extension of the fluid analogy to account for the higher order dynamics brought about by the use of second derivatives is shown in Figure 6. Here the action of a feedback controller acting on measured values of the inventory and controlling a reversible pump would potentially result in a richer variety of dynamic responses that cannot be observed from the first-order model according to Equation (29).

For illustrative purposes, we show in Figure 7 the dynamic response of the fluid analogy for TPB after a step change on the variable  $\xi_1$  for the inflow to the attitude inventory in the case of first-order models (dashed red line) and second-order models (dash-dotted blue line). We consider two different interventions changing the values of the strength of beliefs and evaluation of the outcomes from  $\{b_1 = 4, e_1 = 2\}$ , that is,  $\xi_1 = 8$  to  $\{b_1 = 6, e_1 = 6\}$ , that is,  $\xi_1 = 36$  for the first-order case and  $\{b_1 = 8, e_1 = 8\}$ , that is,  $\xi_1 = 64$ , for the second-order case, respectively. The other two exogenous variables are left constant at  $\xi_2 = \xi_3 = 1$ . We assume no time delays for the exogenous variables ( $\theta_1 = \dots = \theta_3 = 0$ ). A time delay of 2 days is assumed for the inflows on both intention and behaviour tanks ( $\theta_4 = \dots = \theta_8 = 2$ ). The inventories for attitude, subjective norm and PBC have the same time constant  $\tau_1 = \tau_2 = \tau_3 = 1$  day, whereas the inventories for intention and behaviour have time constants of  $\tau_4 = 2$  and  $\tau_5 = 4$  days, respectively. The exogenous inflow resistances are set to unity ( $\gamma_{ij} = 1$ ), whereas the outflow resistance for all inventories is  $\beta_{ij} = 0.5$ . This means that only half of the outflow serves as an inflow to the next inventory in the series. No disturbances are considered in this simulation, that is,  $\zeta_i = 0$ . For the second-order system, the damping coefficient is  $\zeta = 0.3$  for all inventories, whereas  $\tau_a = -3$  for the attitude inventory and  $\tau_a = 0$  for the other inventories. The responses in Figure 7 demonstrate that even with a low-order system a diverse series of dynamical responses can be obtained, among these overdamped (dashed red line, all inventories), underdamped (dash-dotted blue line, all inventories) and inverse response (attitude inventory, dash-dotted blue line). The lag between the inventories is reflected in increasingly longer settling times and consequently slower dynamics between attitude, intention and behaviour.

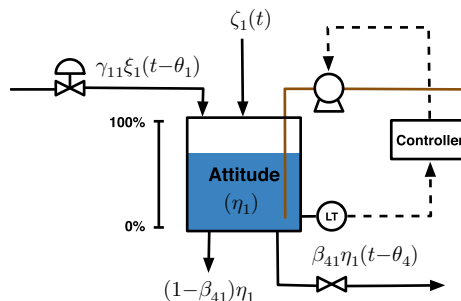


Figure 6. Fluid analogy for representing attitude in a dynamic TPB model using second-order derivatives.

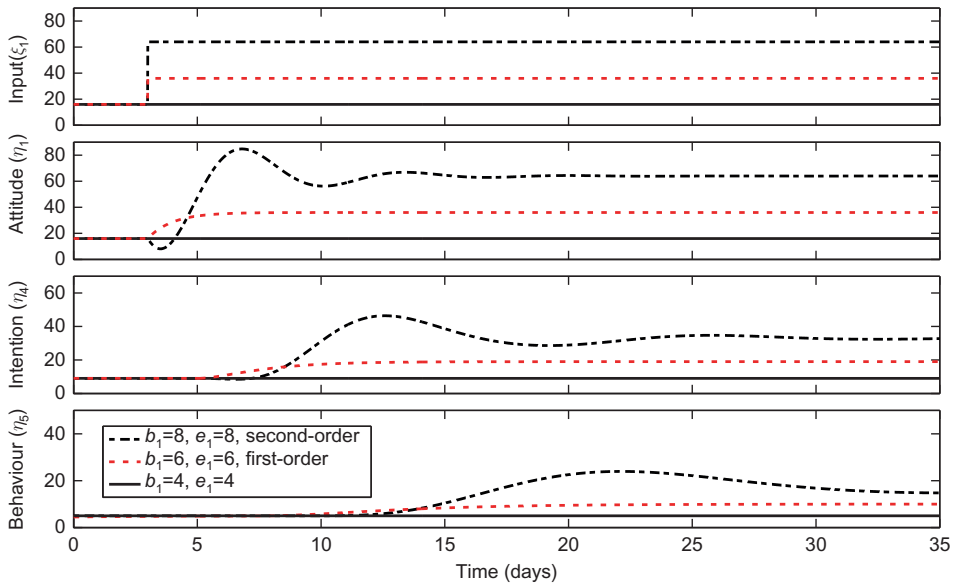


Figure 7. Step response for the dynamic fluid analogy of the TPB for different  $\xi_1 = b_1 \times e_1$  values, and contrasting first-order (dashed red line) versus second-order (dash-dotted blue line) responses. For the second-order system, the damping coefficient is  $\zeta = 0.3$  for all inventories.  $\tau_a = -3$  for the attitude inventory and  $\tau_a = 0$  for the other inventories. Additional parameters are  $\xi_2 = \xi_3 = 1, \theta_1 = \dots = \theta_3 = 0, \theta_4 = \dots = \theta_8 = 2, \tau_1 = \tau_2 = \tau_3 = 1, \tau_4 = 2, \tau_5 = 4, \gamma_{ij} = 1, \beta_{ij} = 0.5$  and  $\zeta_i = 0$ .

#### 4. Simulation study

The overall dynamical model for the behavioural intervention integrates the energy balance model described in Section 2.2 and the dynamic fluid analogy for the TPB described in Section 3.2 by having the output of two distinct TPB models (representing physical activity and diet behaviours, respectively) serving as inputs to the mechanistic energy balance model. This enables the impact of the intervention to be observed in both psychological and physical outcome variables over time. Figure 8 expands on the general system diagram shown in

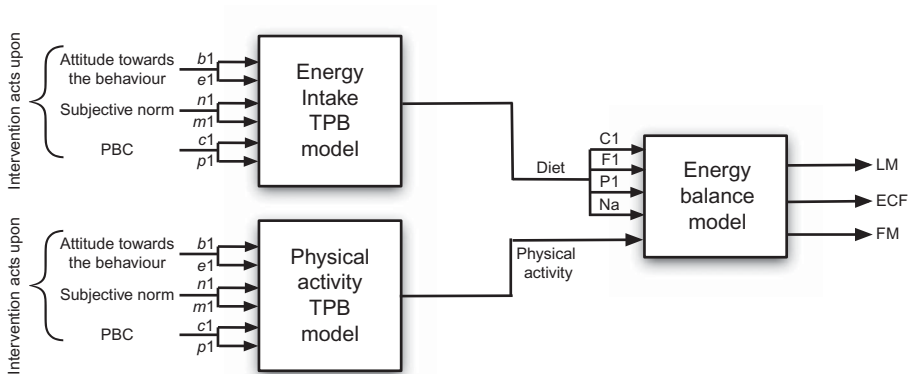


Figure 8. The dynamical model for behavioural interventions integrates the energy balance model described in Section 2.2 and the dynamic fluid analogy for the TPB described in Section 3.2.

Figure 1. The inputs for the behavioural models are the intervention dosage levels applied to the three components of the TPB: attitude towards behaviour (the strength of the beliefs about the outcome  $b_1$  and the evaluation of the outcome  $e_1$ ), subjective norm (the normative beliefs  $n_1$  and the motivation to comply  $m_1$ ) and the PBC (strength of control belief  $c_1$  and the perceived power of control  $p_1$ ). The outputs of the behavioural model, namely, the changes on eating habits and exercise, are translated into changes in diet and physical activity, respectively, which are the inputs for the energy balance model. For the three-compartment model, diet is subdivided into four different signals: carbohydrate intake (CI), fat intake (FI), protein intake (PI) and sodium intake (Na). The outputs of the energy balance model are the fat mass (FM), the lean tissue mass (LM) and the extracellular fluid mass (ECF).

In this section, we present two simulation scenarios that depict some of the useful ways in which behavioural scientists could utilize the proposed intervention model. The first study (Section 4.1) is a participant-focused scenario where the main goal is to analyse the different behavioural responses that could be obtained for a fixed intervention, given different parameter values among participants. The second study (Section 4.2) is an intervention-focused scenario where the main goal is to use the model to examine the order of intervention components and thus optimize the intervention for a particular individual.

#### 4.1. Understanding participant variability through simulation

The simulation study in this section consists of examining the effects over time of an intervention that promotes healthy eating habits and increased physical activity for a representative male participant with the following initial conditions: BM = 100 kg, FM = 30 kg, FFM = 45 kg and ECF = 25 kg. The initial energy intake is EI = 3500 kcal/d, where the diet consists of carbohydrates CI = 482 g, fat FI = 124 g and protein PI = 112 g. The initial physical activity coefficient is  $\delta = 1$  kcal/kg/d. Figure 9 (top) shows the responses of the intervention on TPB models for energy intake behaviour (EI-TPB) and physical activity behaviour (PA-TPB). Figure 9 (bottom) shows the changes in the body compartments corresponding to these interventions. We consider a scenario in which because of the intervention the intensity of beliefs about healthy eating habits increases from  $b_1 = 7$  to  $b_1 = 10$ . This change leads to an increase on the exogenous variable  $\xi_1$  in the EI-TPB system. In the same manner, we assume that because of the intervention there is a change in the beliefs about proper exercising from  $b_1 = 1$  to  $b_1 = 3$ , which also leads to an increase on the variable  $\xi_1$  but in the PA-TPB system. For simplicity, no outflow from the inventory PBC to the inventory behaviour is considered for both behavioural models, that is,  $\beta_{53} = 0$ .

For this simulation study we consider the following three sub-scenarios:

- (i) The participant fully assimilates the intervention and almost immediately starts improving eating habits and engaging in exercise. This means a rapid time constant in the attitude inventory ( $\tau_1 = 0.1$ ), no depletion in the intention inventory ( $\beta_{41} = 1$ ) and no delay in the behaviour inventory ( $\theta_7 = 0$ ).
- (ii) The participant partially and slowly assimilates the intervention. This is represented by a large time constant in the attitude ( $\tau_1 = 20$  days), depletion on intention of  $\beta_{41} = 0.5$  (only 50% of the outflow makes the next inventory) and a delay in behaviour of  $\theta_7 = 15$  days.
- (iii) The same scenario as for Case (ii) but with disturbances on the attitude towards healthy eating and exercising. These disturbances are represented as white noise signals  $\zeta_1 \sim N(0, 20)$  and  $\zeta_1 \sim N(0, 50)$  for the energy intake and physical activity TPB models, respectively.

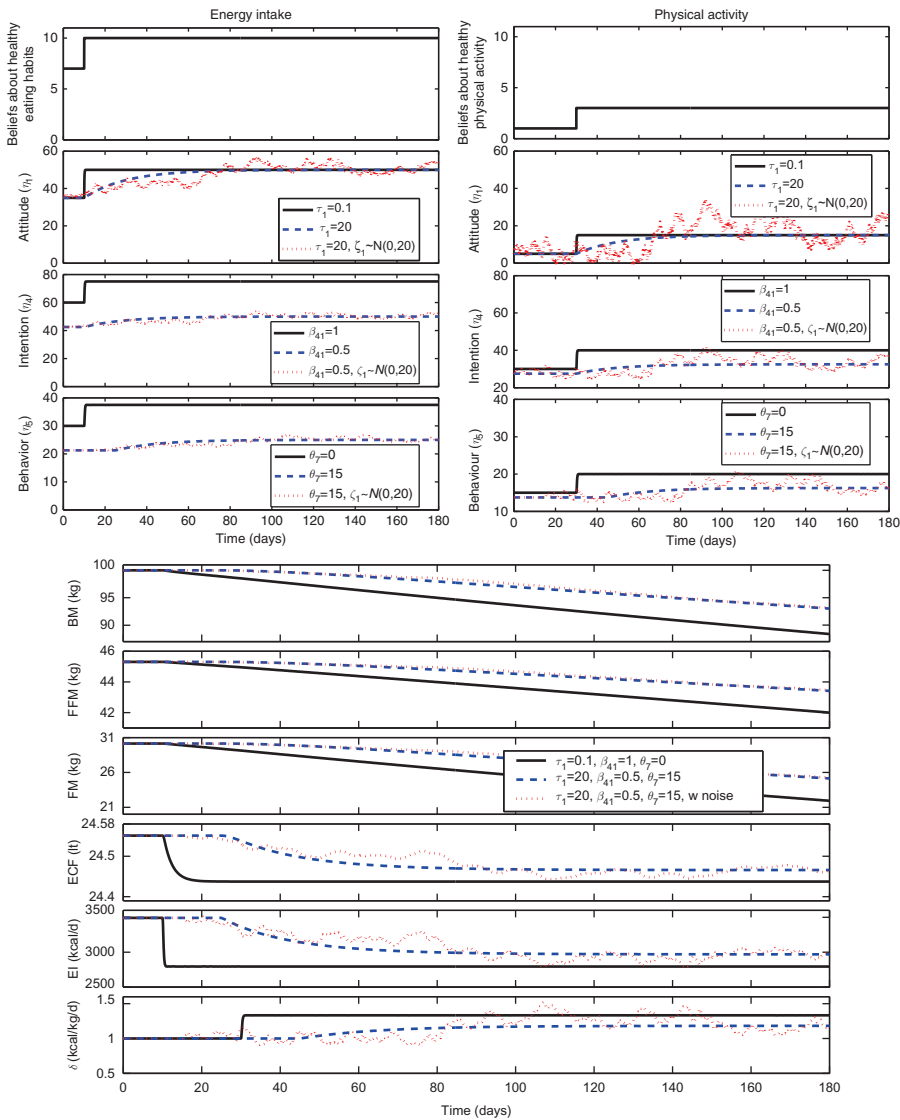


Figure 9. (top) Responses for the energy intake behaviour (EI-TPB) and physical activity behaviour (PA-TPB) models for interventions influencing beliefs about the outcome  $b_1$ ; (bottom) changes in body compartments and total effect of intervention on EI and PA. Simulations for the following intervention cases: (i) complete assimilation  $\tau_1 = 0.1, \beta_{41} = 1, \theta_7 = 0$ , (ii) partial assimilation  $\tau_1 = 20, \beta_{41} = 0.5, \theta_7 = 15$  and (iii) partial assimilation as in Case (ii) but with noise  $\zeta_1 \sim N(0, 20)$  (for EI), and  $\zeta_1 \sim N(0, 50)$  (for PA). Additional parameters:  $\xi_2 = \xi_3 = 50, \theta_1 = \dots = \theta_6 = 0, \theta_8 = 0, \tau_2 = \dots = \tau_5 = 0.1, \gamma_{ij} = 1, \beta_{43} = \beta_{54} = 0.5$  and  $\beta_{53} = 0$ .

After a step change in  $\xi_1$  is introduced, changes are observed in the magnitude level of the inventories for attitude, intention and behaviour, respectively. Figure 9 shows that the attitude response in scenario (ii) takes a larger number of time steps to reach the steady state compared with scenario (i). This occurs because of the different  $\tau_1$  values; the larger value of



$\tau_i$  represents slower dynamics and a longer transition of the system to the new steady state. Moreover, the level of intention in scenario (ii) is much lower than in scenario (i). The smaller the value for  $\beta_{ij}$ , the larger the depletion. Finally, the change in behaviour in scenario (ii) starts much after the change in behaviour in scenario (i). The larger the value for  $\theta_i$ , the longer the delay.

A number of conclusions can be drawn from these simulation results. The most significant is the contrast between the results of Case (i) versus Case (ii) after a six-month time period. The weight for the participant in Case (i) decreases by almost 10 kg over the six months, whereas for Case (ii) the weight loss is only 7 kg. The behavioural ‘lag’ has resulted in a substantially lower achievable weight loss for the participant. However, despite the presence of stochastic disturbances in Case (iii) and correspondingly large fluctuations on the amplitude for some of the inventory levels in the TPB models, these do not result in significant differences on total body weight loss compared with Case (ii).

#### 4.2 Intervention optimization through simulation

The simulation study in this section consists of using the dynamical model summarized in Figure 8 to examine the effects of changing facets of the intervention for an individual or group displaying fixed characteristics. The term ‘optimization’ is used loosely as formal optimization methods will not be applied; rather we present a case in which the model informs the user on the proper order of intervention components for a given participant (or participant group). We examine the effects over time of two different interventions promoting healthy eating habits and increased physical activity for a representative female participant, 1.70m tall, at the following initial conditions: BM = 80 kg, FM = 28 kg, LM = 32 kg and ECF = 20 kg. The initial energy intake is EI = 3500 kcal/d, where the initial diet consists of CI = 482 g, FI = 124 g, and PI = 112 g. The initial physical activity coefficient is  $\delta = 1$  kcal/kg/d. Table 2 shows the time constants  $\tau_i$  and time delays  $\theta_i$ , and Table 3 shows the gains assumed for the participant, respectively. The gains  $\beta_{ij}$  for behavioural eating habits and exercising have been taken from the literature [32,35].

The assumption in the two interventions cases is that each accomplishes equivalent changes to the inputs of the energy intake (EI-TPB) and physical activity (PA-TPB) TPB models; these changes are listed in Table 4. The order in which these changes are introduced differ, as summarized below:

Table 2. Time constants  $\tau_i$  and time delays  $\theta_i$  assumed for female participant in simulation study for different intervention order shown in Figure 10.

Parameter	EI-TPB	PA-TPB
$\tau_1$	20	30
$\tau_2$	30	30
$\tau_3$	10	10
$\tau_4$	10	20
$\tau_5$	30	30
$\theta_1, \dots, \theta_3$	0	0
$\theta_4, \dots, \theta_6$	5	10
$\theta_7$	20	20
$\theta_8$	15	20

Table 3. Inflow and outflow gains  $\gamma_{ij}, \beta_{ij}$  assumed for female participant in simulation study for different intervention order shown in Figure 10.

Parameter	EI-TPB	PA-TPB
$\gamma_{11}$	0.5	0.7
$\gamma_{22}$	0.3	0.5
$\gamma_{33}$	0.7	0.7
$\beta_{41}$	0.16	0.34
$\beta_{42}$	0.04	0.27
$\beta_{43}$	0.59	0.13
$\beta_{53}$	0.1	0.08
$\beta_{54}$	0.32	0.42

Table 4. Behavioural changes on diet and exercising for simulation study for different intervention order shown in Figure 10.

Parameter	EI-TPB	PA-TPB
$b_1$	7 → 8	1 → 5
$e_1$	6 → 7	4 → 7
$s_1$	2 → 4	7 → 9
$m_1$	3 → 4	8 → 10
$p_1$	1 → 2	4 → 10
$c_1$	2 → 3	2 → 8

- (i) *Intervention sequence A.* For energy intake, the intervention components addressing subjective norms begin at day  $t = 30$ , whereas those influencing PBC and attitude occur at day  $t = 60$  and day  $t = 90$ , respectively. For physical activity, intervention components influencing subjective norms enter at day  $t = 10$ , whereas those addressing PBC and attitude occur at day  $t = 60$  and day  $t = 90$ , respectively. This intervention sequence is represented with solid curves in Figure 10.
- (ii) *Intervention sequence B.* For energy intake, the intervention components addressing PBC begin at day  $t = 30$ , whereas those influencing attitude and subjective norms occur at day  $t = 60$  and day  $t = 90$ , respectively. For physical activity, intervention components influencing PBC enter at day  $t = 10$ , whereas those addressing attitude and subjective norms occur at at day  $t = 60$  and day  $t = 90$ , respectively.

Figure 10 (top) shows the participant response for the EI-TPB and PA-TPB models, whereas Figure 10 (bottom) shows the changes in body composition corresponding to these interventions. Sequence A results in 10 kg total weight loss after 6 months, whereas Sequence B accomplishes 15 kg total weight loss in the participant during the same time period.

The simulation results indicate that intervention sequence B represents a more suitable alternative for the participant under study than sequence A. Figure 10 (top) shows that improved outcomes are obtained in intervention sequence B by placing a priority in the intervention on PBC in lieu of subjective norms. This action leads to faster changes in behaviour for both energy intake and physical activity, in contrast to intervention sequence A.

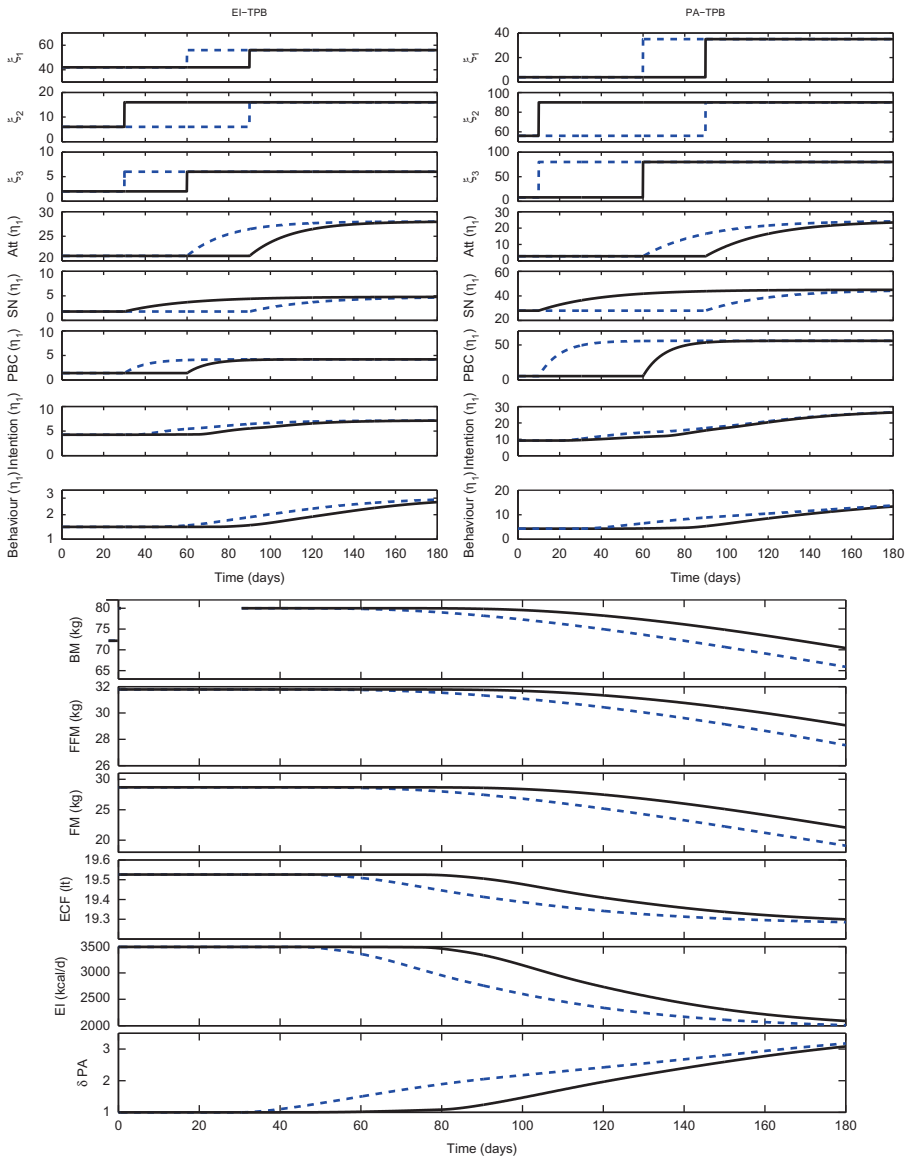


Figure 10. (top) Responses for the energy intake behaviour (EI-TPB) and physical activity behaviour (PA-TPB) models for two intervention sequences; (bottom) changes in body compartments and total effect of the intervention on EI and PA. Simulations for the following intervention cases: intervention sequence A (solid), leading to a decrease on weight of 10 kg in six months, and intervention sequence B (dashed), leading to a weight loss of 15 kg during the same time period. Additional parameters are shown in Tables 2–4.

This occurs because for the particular gain values  $\gamma_{ij}$  and  $\beta_{ij}$ , which define this participant, it is more prudent to apply first an intervention that affects PBC and attitude, rather than subjective norms. Moreover, the levels of these inventories reach a steady state much faster than for the subjective norm inventory. Finally, because the inventory for PBC has two

outflows (one flowing into the intention inventory, and the other flowing into behaviour), a larger constant inflow into the inventory of PBC leads to a larger inflow in the inventories for intention and behaviour when using sequence B, in lieu of sequence A.

## 5. Summary and conclusions

A dynamical model for a behavioural intervention associated with weight loss and body change composition has been proposed that provides a potentially useful framework for understanding and optimizing this class of interventions. By testing the effect of intervention components on the outcomes of interest over time, the intervention scientist can optimally decide on aspects of the intervention such as the ordering and strength of the components, and can better predict both the inter- and intra-individual variability that will be reflected in these interventions.

Extensions of this simulation work include incorporating additional forms of random disturbances and model stochasticity to more closely mimic actual responses. An extended version of the dynamic TPB model where the endogenous variables are latent as opposed to observed variables has also been examined; it has not been presented in this article for reasons of brevity.

The simulation results point to the need for data from experimental trials or observational studies that can be used to estimate parameter coefficients in these models and validate the modelling framework. We are currently exploring how methods from the field of system identification [39] and functional data analysis [40], coupled with data resulting from participant diaries or ecological momentary assessment, can be used for this purpose. A long-term goal is to develop adaptive behavioural interventions for preventing weight gain or loss in patients with obesity or malnutrition, relying on control systems engineering principles [6].

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